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$$\binom{n}{k} = \binom{n}{n-k}$$

proof: $\binom{n}{k}$ counts the number of subsets of $\{1, 2, \ldots, n\}$ of size k

$\binom{n}{n-k}$ counts the number of subsets of $\{1, 2, \ldots, n\}$ of size $n-k$

Take a set A inside $\{1, 2, \ldots, n\}$ of size k.

$$A \subseteq \{1, 2, \ldots, n\}$$

Then, A^c is the unique set corresponding to A of size $n-k$

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$X \sim Bin(n, p)$

$$P(X < k) = P_X(0) + P_X(1) + \ldots + P_X(k-1)$$

$$= \sum_{i=0}^{k-1} \binom{n}{i} p^i (1-p)^{n-1}$$

The probability that we get less than k heads after n flips of a coin

$Y \sim NegBin(k, p)$

$$P(Y > n) = P(X < k)$$

$$= \sum_{i=n+1}^{\infty} P_Y(i) = \sum_{i=n+1}^{\infty} \binom{i-1}{k-1} p^k (1-p)^{i-k}$$

probability that it takes more than n flips to get k heads

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

number of sets of size k in $\{1,2,\ldots,n\}$

number of sets of size $k+1$ in $\{1,2,\ldots,n\}$

number of sets of size $k+1$ in $\{1,2,\ldots,n+1\}$

$$\{1,2,\ldots,n\} \cup \{n+1\} = \{1,2,\ldots,n,n+1\}$$

- Take a subset of size $k+1$

case 1: our subset does not have $n+1$ in it

$$\binom{n}{k+1}$$

disjoint

case 2: our subset has $n+1$ in it

$$\binom{n+1-1}{k+1-1} = \binom{n}{k}$$

Thus, $\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$