10/1/25

Recall: - take a probability space (S, P)

- a random variable is a function $X: S \to \mathbb{R}$
- the probability mass function is

$$p_X(k) = P(X = k) \in [0,1]$$

- Bernouli / Indicator random variable (flip a coin 1 time)

$$E \subseteq S, \quad I_E(k) = \begin{cases} 1 & \text{if } k \in E \\ 0 & \text{if } k \notin E \end{cases}$$

$$p_{I_E}(k) = \begin{cases} p & \text{if } k = 1 \\ 1-p & \text{if } k = 0 \end{cases}$$

- Binomial random variables (flipping a coin n times)

X counts the number of H. p is probability of H

$$p_X(k) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

We say that a random variable satisfies a distribution, like "X satisfies the Binomial Distribution".

↳ this means X is a Binomial Random Variable

Notation: $X \sim Ber(p)$ "X is distributed as $Ber(p)$"

↳ X is a Bernouli Random variable

$X \sim Bin(n,p)$ — X is a binomial random variable
n is number of times flip coin
p is probability of getting heads

if $X \sim Ber(p)$, then $X \sim Bin(1,p)$

if $X_1, X_2, \dots, X_n \sim Ber(p)$, then $X_1 + X_2 + \dots + X_n \sim Bin(n,p)$

Geometric Distribution

(flipping a coin until you get heads)

Let X be the random variable that records the spot of the first H if keep flipping a coin.

$$P_X(k) = P(X=k) = (1-p)^{k-1} p$$

$$X \sim Geom(p)$$

Negative Binomial Distribution

(do a Bernouli trial until you win k times)

Let X be the random variable that records the spot of the k-th H.

ex: $k=5$

$s = H\,T\,H\,H\,T\,T\,H\,T\,T\,H\,T\,H\,T\,T \dots$

$s \in S$, $X(s) = 10$

$N = 10$

$$P_X(N) = P(X=N) = P(\{s \in S \mid X(s) = N\})$$

$$= \underbrace{\binom{N-1}{k-1}}\ \underbrace{p^k (1-p)^{N-k}}$$

\# of ways to arrange the K heads among N spots (last H is fixed!)

probability of getting k heads and N-k tails, which can then be oriented a variety of ways

or think about it in reverse order