9/22/25

(on HW 3)
— Review Exercise 1.22: $P(E_2 | E_1^c) \neq 1 - P(E_2 | E_1)$

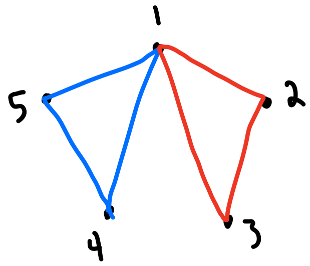
Probabilistic Model

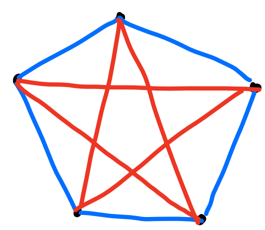
I want to invite people to a party.

At the party, I want 3 people that know each other (each pair), or 3 people that don't know each other.

How many people do I need to invite?

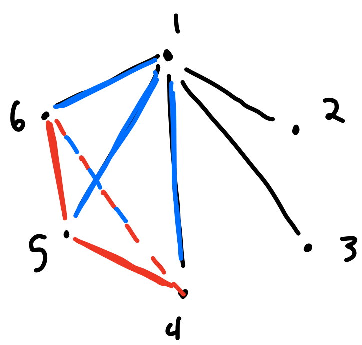
Try 5:





nope!

Answer: 6



- person 1 must either know 3 people, or not know 3 people
- Here, assuming person 1 knows 3 people (w.l.o.g.), we know each of those 3 people must not know eachother, otherwise 3 will know eachother
- But, if all 3 don't know each other, then that group of 3 exists in our party.

$\Rightarrow$ either way, 3 know each other or 3 don't know each other.

Definition (Ramsey Number):
The Ramsey number $R(k,m)$ is the smallest number n, such that if we invite n people to a party, then k people will know each other, or m people won't know each other.

↳ $R(3,3) = 6$

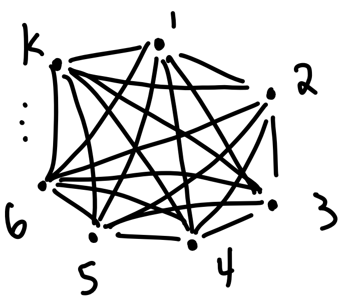
Theorem (Ramsey 1929): $R(k,m)$ is finite.

- if $R(k,m)$ is finite, then how can we compute it?
  can we give bounds?

Theorem (Erdős): If $\binom{n}{k} \cdot 2^{1-\binom{k}{2}} < 1$, then $R(k,k) > n$.

Thus, $R(k,k) > \lfloor 2^{\frac{k}{2}} \rfloor$

we color the relation red or blue with probability $\frac{1}{2}$, independently. For any set S of k people, let E_S be the event that there are k that know each other (blue) or k that don't know each other (red).



- How many lines are there? $\frac{k(k-1)}{2} = \binom{k}{2}$

either blue or red

$$P(E_S) = 2 \cdot \underbrace{\frac{1}{2} \cdot \frac{1}{2} \cdot \ldots \cdot \frac{1}{2}}_{\binom{k}{2} \text{ times}} = 2^{1-\binom{k}{2}}$$