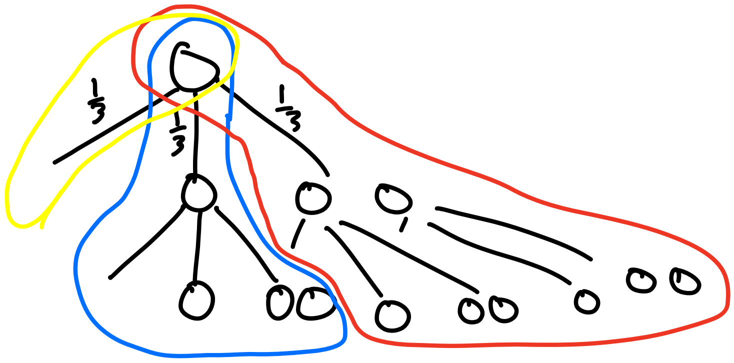
9/17/25

You start with a bacteria and after 10 seconds it dies, stays the same, or splits into 2, with all situations happening with probability $\frac{1}{3}$. After 10 seconds, all remaining and new bacteria behave the same independently. What is the probability that the bacteria will all die eventually?



Let D be the event that all bacteria die eventually. P(D)?

Let B_1 be the event that the first bacteria dies.

Let B_2 be the event that the first bacteria stays the same.

Let B_3 be the event that the first bacteria splits in two.

* $P(B_1) = \frac{1}{3}$, * $P(B_2) = \frac{1}{3}$, * $P(B_3) = \frac{1}{3}$

Observation: B_1, B_2, B_3 partition the sample space

recall: Law of total probability:

$$P(D) = \underbrace{P(D \mid B_1)}_{1}\underbrace{P(B_1)}_{\frac{1}{3}} + \underbrace{P(D \mid B_2)}_{P(D)}\underbrace{P(B_2)}_{\frac{1}{3}} + \underbrace{P(D \mid B_3)}_{P(D) \cdot P(D)}\underbrace{P(B_3)}_{\frac{1}{3}}$$

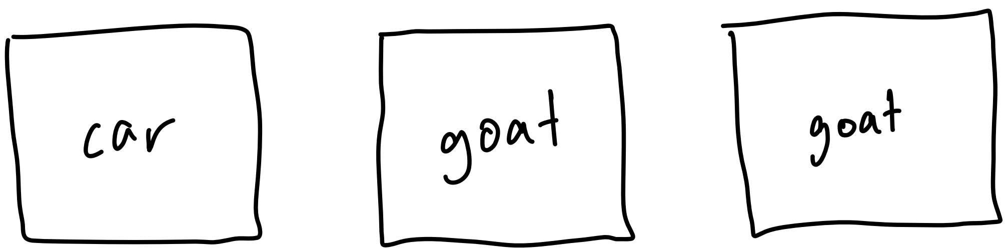
$$P(D) = \frac{1}{3} + \frac{1}{3}P(D) + \frac{1}{3}P(D)^2$$

$$0 = \frac{1}{3} - \frac{2}{3}P(D) + \frac{1}{3}P(D)^2$$

$$0 = 1 - 2P(D) + P(D^2)$$

$$0 = (P(D) - 1)^2 \implies P(D) = 1$$

The Monty Hall Problem

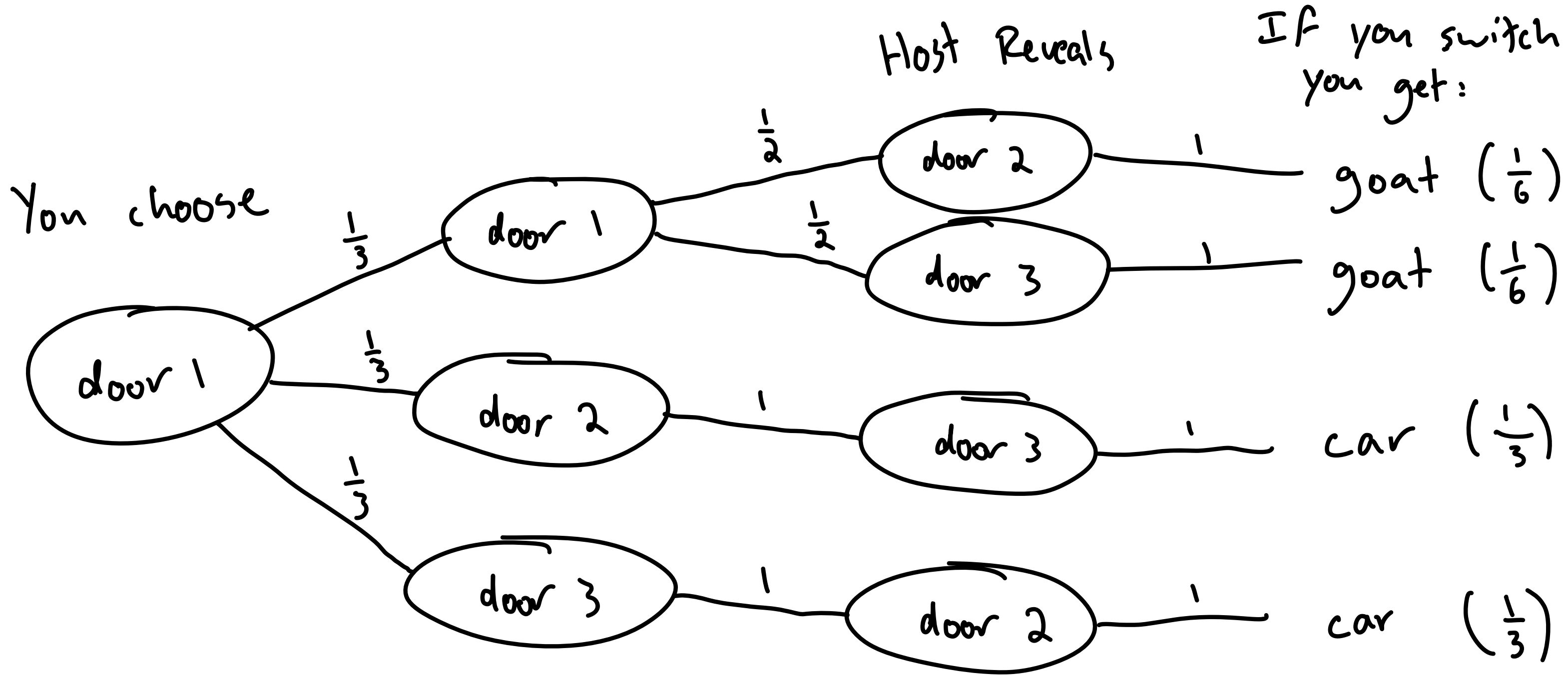


1. Pick a door (you don't know what is behind each door)
2. Host reveals one of the doors that you didn't pick
   ↳ always reveals goat; if you chose car, host picks at random
3. Host asks if you want to switch doors

Do you switch?

↳

$\Omega = \{E_{1,2}, E_{1,3}, E_{2,3}, E_{3,2}\}$ where $E_{i,j}$ represents the event that, given an initial choice of (an arbitrary) door 1, the car is behind door i and the host reveals door j.



$\Rightarrow P(E_{1,2}) = P(E_{1,3}) = \frac{1}{6}$

$P(E_{2,3}) = P(E_{3,2}) = \frac{1}{3}$

If you switch,

$P(\text{car}) = \frac{2}{3}$:)

If you don't switch,

$P(\text{car}) = \frac{1}{3}$:(