9/8/25

Last class: How many ways to permute LALALAA?

$$\binom{7}{4} = \binom{7}{3} = \frac{7!}{4!\,3!}$$

what about the word STATISTICS?

$$\frac{10!}{3!\,3!\,2!} = \underset{\text{spots for S}}{\binom{10}{3}} \underset{\text{spots for T}}{\binom{7}{3}} \underset{\text{spots for I}}{\binom{4}{2}} \underset{\text{spot for A}}{\binom{2}{1}} \underset{\text{spot for C}}{\binom{1}{1}}$$

we need an integer n.

Then we need integers $n_1, n_2, \ldots, n_k$

we also want $n_1 + n_2 + \cdots + n_k = n$

"what is the number of permutations of a word of length n where the word has n_1 letters of a certain type, n_2 letters of a certain type, . . . , and k letters of a certain type?"

$$\frac{n!}{n_1!\,n_2!\cdots n_k!} = \binom{n}{n_1}\binom{n-n_1}{n_2}\cdots\binom{n-(n_1+n_2+\cdots+n_{k-1})}{n_k}$$

Definition (Multinomial Coefficient):

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1! n_2! \cdots n_k!} = \binom{n}{n_1}\binom{n-n_1}{n_2} \cdots \binom{n-(n_1+n_2+\dots+n_{k-1})}{n_k}$$

Multinomial Theorem:

$$(x_1 + x_2 + \dots + x_k)^n = \sum \binom{n}{e_1, e_2, \dots, e_k} x_1^{e_1} x_2^{e_2} \cdots x_k^{e_k}$$

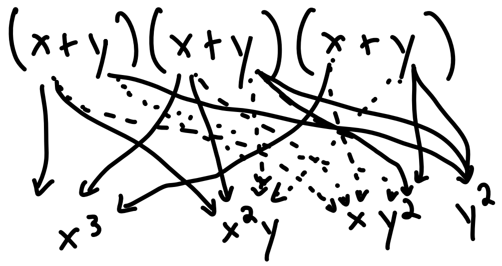
$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \binom{n}{k, n-k}$$

Recall: Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

note: $\binom{n}{k} = \binom{n}{n-k}$

Intutitive Example:



y^3 : only 1 way to combine 0 x's: $\binom{3}{0} = 1$

xy^2 : pick 1 x among 3, rest give y's : $\binom{3}{1} = 3$

x^2y : pick 2 x's among 3, rest give y : $\binom{3}{2} = 3$

x^3 : only 1 way to combine 3 x's: $\binom{3}{3} = 1$

$$\Longrightarrow (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

A box contains 10 red balls, 10 white balls, and 10 blue balls. Take out 5 balls without replacement. what is the probability that you do not get every color?

Let this event be E.

Let R be the event that we don't have any red balls

Let B be the event that we don't have any white balls

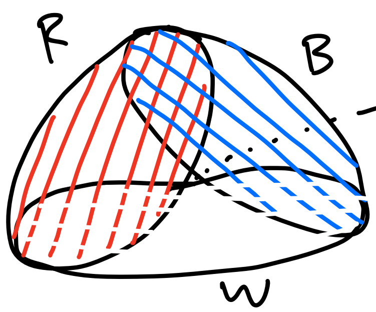
Let W be the event that we don't have any blue balls

$$E = R \cup B \cup W$$

not disjoint! (two could be true at the same time)

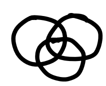
$$P(E) = P(R \cup B \cup W)$$

$$= P(R) + P(B) + P(W) - P(R \cap B) - P(R \cap W) - P(B \cap W)$$



$+ P(A \cup B \cup W)$

(doesn't exist)

but could in other situations, e.g. 

$$P(R \cap B) = \frac{\binom{10}{5}}{\binom{30}{5}} = P(R \cap W) = P(B \cap W)$$

only white to choose from

$$P(R) = \frac{\binom{20}{5}}{\binom{30}{5}} = P(W) = P(B)$$

B+W to choose from

$$P(E) = P(R \cup B \cup W) = \frac{3 \cdot \binom{10}{5}}{\binom{30}{5}} - \frac{3 \cdot \binom{20}{5}}{\binom{30}{5}} + 0$$

★ Inclusion - Exclusion Principle