9/5/25

Probability Class: - Pure Math

how do we map between the two?

- Applications of the pure math

What is the probability of a single person being born on a certain day of the year?

1. What can I assume?
   - I want all days equally likely
   - Make all years have 365 days

2. Construct a probability space for the problem:

$$S = \{1, 2, 3, \ldots, 365\}$$

sample space, a simplified way of viewing all outcomes

$$E \subseteq S, \quad P(E) = \frac{|E|}{|S|}$$

- Let E be the event that a person is born in the first 146 days of the year.

$$P(E) = \frac{|E|}{|S|} = \frac{|\{1,2,3,\ldots,146\}|}{365} = \frac{146}{365}$$

✱ what are the consequences of the model we <u>chose</u> to construct?

Suppose some days of the year are more likely for birth than others. How would we adjust the model?

- Say we want $P(\text{Jan } 1^{st})$ to be double, but everything else stays the same

MODIFY MODEL

$S = \{1, 2, 3, \ldots, 365\}$ $\quad E \subseteq S$

$W = S \rightarrow [0, \infty)$ $\quad w(1) = 2,\ w(2) = 1,\ w(3) = 1,\ \ldots$

$$P(E) = \frac{\sum_{a \in E} w(a)}{\sum_{a \in S} w(a)}$$

Suppose now we want to take k people and find the probability that 2 people have the same birthday?

MODIFY MODEL

want S = the set of all lists of length k, whose entries are at least 1 and at most 365.

$S_1 = \{1, 2, 3, \ldots, 365\}$ $\quad S_2 = \{1, 2, 3, \ldots, 365\}$ $\quad S_3 = \{1, 2, 3, \ldots, 365\}$

cartesian product

$$S = S_1 \times S_2 \times S_3$$

$E \subseteq S$ $\quad P(E) = ?$

Let E be the event that person 2 is born on day 55.

$$P(E) = \frac{1}{365}$$ (assume you want this)

$$E_1 \subseteq S_1, \; E_2 \subseteq S_2, \; \cdots, \; E_k \subseteq S_k$$

$$P(E_1 \times E_2 \times E_3 \times \cdots \times E_k) = P(E_1)P(E_2)P(E_3)\cdots P(E_k)$$

(?)

→ shows that there is a unique probability function (follows intuition)

what is the probability that 2 people share a birthday?

↓ (first let's switch the question)

what is the probability that no two people share a birthday?

↳ goal: count the number of lists with no repetitions.

$$= (365)_k = (365)(364)\cdots(365-k+1)$$

probability is $\dfrac{(365)_k}{365^k}$ (complement)

$\Rightarrow$ probability of 2 shared birthdays is $1 - \dfrac{(365)_k}{365^k}$

Moving on:

How many ways are there to permute the word:

LALALAA ?

$$\binom{7}{4} = \binom{7}{3} = \frac{7!}{4!\,3!}$$

why not just 7! ?

because we don't care how the L's or A's are ordered among themselves.