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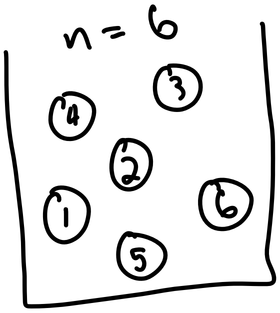
- probability is a way of measuring spaces

Finite probability is just counting.

Say we have a box with n balls $b_1, b_2, \ldots, b_n$



Let K be a natural number.

We do the following:

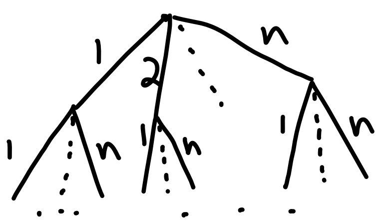
1. Take a ball out, record its number
   Put the ball back in the box (shake the box)
2. Take a ball out, record its number
   Put the ball back in the box (shake the box)
3. (continue for a total of K times)

↳ generates $m_1 m_2 m_3 \cdots m_K$

Q: How many lists can arise from this procedure?

A: if $n=6$, 6^K ; generally n^K

Multiplication Principle:

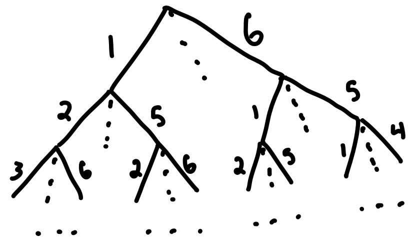


important here:

* we are <u>replacing the ball each time</u>
* we are <u>recording the order</u>

what if we don't replace the balls, and do record the order:

$n = 6, \quad k = 3$



$6 \cdot 5 \cdot 4$

generally: $n(n-1)(n-2)\cdots(n-k+1)$

Falling factorial: $(n)_k = n(n-1)(n-2)\cdots(n-k+1) = \dfrac{n!}{(n-k)!}$

$$(n)_n = n(n-1)(n-2)\cdots(n-n+1) = n!$$

Don't replace the ball, don't record the order:

$n = 5, \quad k = 3$

ex: $\left.\begin{matrix} 154 \\ 145 \\ 415 \\ 451 \\ 514 \\ 541 \end{matrix}\right\}$ equivalent

$$\frac{(n)_k}{k!} = \binom{n}{k}$$

"n choose k" / Binomial Coefficient – the number of ways you can pick a group of k things out of n things, where order does not matter

Binomial Theorem:

if x, y are real numbers, then

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$