4/2/25

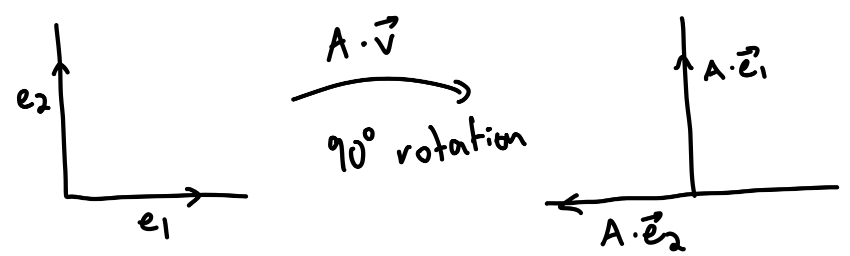
Ex: $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Find e-vals & e-vecs

~HW asks for real eigenvalues (none)

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$$

$$\lambda^2 = -1 \qquad \lambda = \pm i$$

what is A doing?




No eigenvectors! (in real plane)

(challenge from last class)

Ex: $A$ $2\times 2$, $A\vec{v}_1 = -\vec{v}_1$, $A\vec{v}_2 = 2\vec{v}_2$

$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\qquad A^5 = ?$

Soln To find a $2\times 2$ matrix $B$, we need $B\vec{e}_1$, $B\vec{e}_2$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \underline{?}\,\vec{v}_1 + \underline{?}\,v_2 \qquad \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{?}\,\vec{v}_1 + \underline{?}\,\vec{v}_2$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|cc} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

$\begin{bmatrix}1\\0\end{bmatrix} = -3\vec{v}_1 + 2\vec{v}_2 \qquad \begin{bmatrix}0\\1\end{bmatrix} = 2\vec{v}_1 - \vec{v}_2$

$$A^5\begin{bmatrix}1\\0\end{bmatrix} = A^5(-3v_1 + 2v_2) = -3A^5v_1 + 2A^5v_2$$

$$= -3(-1)^5 v_1 + 2(2)^5 v_2$$

$A\vec{v}_1 = (-1)\vec{v}_1$

$A^5\vec{v}_1 = (-1)^5 v_1$

$$= 3\begin{bmatrix}1\\2\end{bmatrix} + 64\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}3\\6\end{bmatrix} + \begin{bmatrix}128\\192\end{bmatrix}$$

$$= \begin{bmatrix}131\\198\end{bmatrix}$$

$$A^5\begin{bmatrix}0\\1\end{bmatrix} = A^5(2v_1 - v_2) = 2A^5v_1 - A^5v_2$$

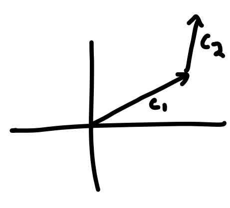
$$= 2(-1)^5 v_1 - (2)^5 v_2$$

$$= -2\begin{bmatrix}1\\2\end{bmatrix} - 32\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}-2-64\\-4-96\end{bmatrix} = \begin{bmatrix}-66\\-100\end{bmatrix}$$

$$A^5 = \begin{bmatrix}131 & -66\\198 & -100\end{bmatrix}$$

## Change of Basis

<u>Ex:</u> $\beta = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^2$




Given $\vec{v} = c_1\vec{v}_1 + c_2v_2 \qquad [v]_\beta = \begin{bmatrix}c_1\\c_2\end{bmatrix}$

<u>Prop</u>: Given a basis $\beta = \{v_1, \dots, v_n\}$ for $\mathbb{R}^n$,

the fn $T: \mathbb{R}^n \to \mathbb{R}^n$ w/ $T(\vec{v}) = [\vec{v}]_\beta$

is a linear transformation

Pf $\boxed{+}$ $\vec{u} = c_1\vec{v}_1 + \dots + c_n\vec{v}_n$ , $\vec{v} = b_1\vec{v}_1 + \dots + b_n\vec{v}_n$

$$\vec{u}+\vec{v} = (c_1+b_1)\vec{v}_1 + \dots + (c_n+b_n)\vec{v}_n$$

$$T(\vec{u}+\vec{v}) = \begin{bmatrix} c_1+b_1 \\ \vdots \\ c_n+b_n \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = T(\vec{u}) + T(\vec{v})$$

$\boxed{\cdot}$ $T(c\vec{u}) = \begin{bmatrix} cc_1 \\ \vdots \\ cc_n \end{bmatrix} = c\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = cT(\vec{u})$

$\square$

Q: what is the matrix for this T ?

A is matrix $\left[ T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \; T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) \right]$

$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}-3\\2\end{bmatrix}$ $\quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}2\\-1\end{bmatrix}$

$A = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$ which is inverse of $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ !