3/19/25

- Quiz recap (look at T/F online)

Two ways to compute $\det A$:

- Put in RREF (and keep track of what operations do to $\det A$)

- Cofactor expansion: $C_{ij} = (-1)^{i+j} A_{ij}$

  good for small or sparce matrices

  $$\det A = a_{i1} C_{i1} + \dots + a_{in} C_{in}$$

Ex: Compute $\det A$, $A = \begin{bmatrix} 5 & -2 & 4 & -1 \\ 0 & 1 & 5 & 2 \\ 1 & 2 & 0 & 1 \\ -3 & 1 & -1 & 1 \end{bmatrix}$

straight lines means det[ ]

$$\det A \overset{R_1 \leftrightarrow R_2}{=} - \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 5 & 2 \\ 5 & -2 & 4 & -1 \\ -3 & 1 & -1 & 1 \end{vmatrix} \overset{\substack{R_3 - 5R_1 \\ R_4 + 3R_1}}{=} - \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & -12 & 4 & -6 \\ 0 & 7 & -1 & 4 \end{vmatrix}$$

$$\overset{\substack{R_3 + 12R_2 \\ R_4 - 7R_2}}{=} - \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 64 & 18 \\ 0 & 0 & -36 & -10 \end{vmatrix} \overset{R_4 + \frac{36}{64} R_3}{=} - \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 64 & 18 \\ 0 & 0 & 0 & -10 + \frac{9 \times 18}{16} \end{vmatrix}$$

$$= -64 * \left(-10 + \frac{9 \times 18}{16}\right)$$

$$= 640 - 36 * 18$$

$$= 640 - 648$$

$$= -8$$

Facts: A, B $n \times n$

- $\det(AB) = \det A \det B$
- $\det A^T = \det A$

Cor: You can compute determinants by expanding along any row or column.

Ex:
$$\begin{vmatrix} 3 & 2 & 0 & 1 & 3 \\ -2 & 4 & 1 & 2 & 1 \\ 0 & -1 & 0 & 1 & -5 \\ -1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 3 & 2 & 0 & 1 \\ -2 & 4 & 1 & 2 \\ 0 & -1 & 0 & 1 \\ -1 & 2 & 0 & -1 \end{vmatrix} = 2(-1) \begin{vmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ -1 & 2 & -1 \end{vmatrix}$$

$$= -2\left(3\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}\right) = -2(3(-1) - 1(3))$$

$$= -2(-6) = 12$$