2/5/25 ☆ Exam 1 next Friday (no quiz next week)

**Recap:** Elementary Matrices

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix}$$

$\uparrow$
$R_1 + R_2$

**Corollary:** If $A$ can be row reduced to $H$, then we can find elementary matrices $E_1, \dots, E_k$ w/ $H = E_k E_{k-1} \dots E_1 A$

Ex: $A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}$ $\quad H = RREF(A)$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 4R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = H = I$$

$$I = \underbrace{\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}}_{R_1 - 4R_2}\underbrace{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}}_{R_2 - 2R_1}\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{R_1 \leftrightarrow R_2} A$$

(order matters! & $A$ must be on right)

Ex: $A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 9 \\ 1 & -2 \end{bmatrix}$

a) $BA = \begin{bmatrix} -4 & 9 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$ !!

b) Solve $A\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

- can save time by doing work to find B matrix first!

shortcut:

$$BA\vec{x} = B\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$I_2\vec{x} = x = B\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -17 \\ 4 \end{bmatrix}$$

Key: Find a matrix B w/ $BA = I_n$

Def$^n$: Let A be $m \times n$.

- B is a right inverse of A if $AB = I_m$
- B is a left inverse of A is $BA = I_n$
- A square matrix is invertible if there is B w/ $AB = I = BA$. In this case, we write $B = A^{-1}$, and call B the inverse of A
- Only square matrices have inverses

<u>Rmk</u>: $B = A^{-1} \iff B^{-1} = A$

<u>Pf.</u> $B = A^{-1} \implies AB = I = BA \implies B^{-1} = A$

<u>Ex</u>: Show $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ are inverses.

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \&\& \quad \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$