2/24/25

- recap (look at last page from 2/21)

Other vector spaces

Sometimes we study things that are vector spaces but <u>don't</u> live in $\mathbb{R}^n$.

<u>Ex:</u> $S_d = \{\text{polynomials in } x \text{ w/ } \deg \leq d\}$ is a vector space

$x^2+3 \in S_2 \qquad x+1 \in S_2$

Key Properties

1) can add two vectors
2) can scalar multiply

Independence/Dependence/span make sense for abstract vectors

<u>Ex:</u> $x^2+3,\ x+1 \in S_2$ are linearly independent

$c_1(x^2+3) + c_2(x+1) = 0$

$c_1x^2 + 3c_1 + c_2x + c_2 = 0$

$\Rightarrow c_1 = 0 = c_2$

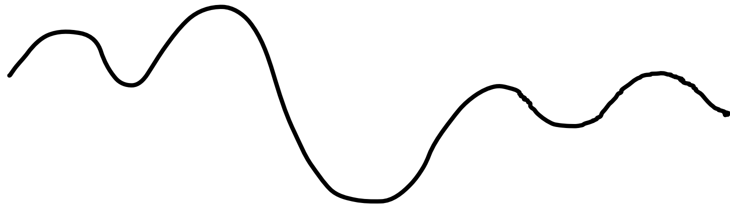
S_2 is 3-dimensional!

<u>Basis for S_2:</u> $1, x, x^2$

<u>Basis for S_d:</u> $1, x, \dots, x^d$

Another Example: $V = \{ f : [0,1] \to \mathbb{R} \text{ w/ } f(0) = f(1) \}$

periodic functions



V is infinite dimensional

Independent vectors: $\sin(2\pi k x)$, $\cos(2\pi k x)$
for k integer

Harmonic Analysis: write any soundwave in terms of $\sin 2\pi k x$ and $\cos 2\pi k x$ "fourier analysis"

Linear Transformations

Defn: A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a function $T : \mathbb{R}^n \to \mathbb{R}^m$ satisfying:

1) $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$ for all $\vec{u}, \vec{v} \in \mathbb{R}^n$
2) $T(c\vec{u}) = cT(\vec{u})$ for all $c \in \mathbb{R}$, $u \in \mathbb{R}^n$

Rmk: Can talk about linear transformations between vector spaces (or subspaces) w/ the same definition.

Prop Let T be a linear transformation.

1) $T(\vec{0}) = \vec{0}$

2) $T(c_1\vec{v}_1 + \dots + c_k\vec{v}_k) = c_1 T(\vec{v}_1) + \dots + c_k T(\vec{v}_k)$

Pf. 1) $T(\vec{0}) = T(0\cdot\vec{0}) = 0\cdot T(\vec{0}) = \vec{0}$

2) $T(c_1\vec{v}_1 + \dots + c_k\vec{v}_k) = T(c_1\vec{v}_1) + T(c_2 v_2 + \dots + c_k v_k)$

$= T(c_1\vec{v}_1) + \dots + T(c_k v_k) = c_1 T(\vec{v}_1) + \dots + c_k T(\vec{v}_k)$ $\square$

Ex: Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ satisfying $T(\vec{v}) = 1.5\vec{v}$

Then T is linear:

✓ 1) $T(\vec{u}+\vec{v}) = 1.5(\vec{u}+\vec{v}) = 1.5\vec{u} + 1.5\vec{v} = T(\vec{u}) + T(\vec{v})$

✓ 2) $T(c\vec{u}) = 1.5(c\vec{u}) = c(1.5\vec{u}) = c\,T(\vec{u})$

Ex: $T: \mathbb{R} \to \mathbb{R}$ $T(x) = x+1$ is <u>not</u> linear

easy way: $T(0) = 0+1 \neq 0$