1/13/25

Def^n: An equation in unknowns $x, y, z, \ldots$ is called a linear equation if both sides of the equation are the sum of constant multiples of the variables plus an optional constant.

Ex:

$x^2 = 1$ not linear, consistent

$x + yz = 3$ not linear, consistent

$5 = 7$ linear, inconsistent

$3x + z = y + \sin 1$ linear, consistent

$3x + z = 1 + \sin y$ not linear, consistent

* by convention, usually arrange to have variables on left and constants on the right

Def^n: A system of linear equations is a collection of linear equations

Ex: $\left.\begin{array}{l} x + 2y = 7 \\ 3x + 2y = 8 \end{array}\right\}$ is a system of linear eqns

Def^n: A solution to a system of linear equations is a collection of values for the variables that satisfy the equation

- A solution set of the system is the set of all solutions

- A system is <u>inconsistent</u> if it has no solutions. It is <u>consistent</u> if it has at least one solution.

<u>Def^n</u>: <u>Solving</u> a system means finding a formula for all solutions

Ex: R1 $x+2y=7$
R2 $3x+2y=8$ $\longrightarrow$ (elimination technique) $-R1+R2$: $2x=1$

$x=\frac{1}{2}$

$\frac{3}{2}+2y=8$

$2y=\frac{13}{2}$ $\quad y=\frac{13}{4}$

are solutions (check)

and are the only solutions

<u>Convention</u>: write a system of equations in an augmented matrix

Ex:

$$\begin{matrix} x+2y=7 \\ 3x+2y=8 \end{matrix} \longrightarrow \begin{array}{c} \begin{matrix} x & y & = \end{matrix} \\ \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 3 & 2 & 8 \end{array}\right] \end{array}$$

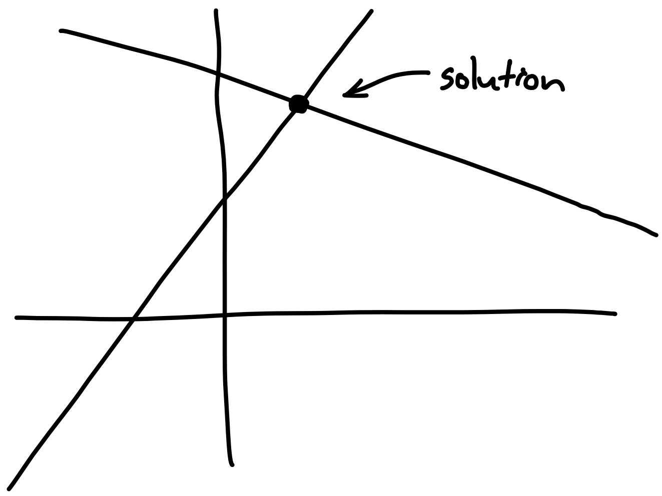
$$\begin{matrix} x+2z=7 \\ 3x-20y=8 \end{matrix} \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 3 & -20 & 0 & 8 \end{array}\right]$$

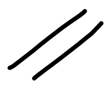
<u>Rmk</u>: Adding and subtracting equations from each other corresponds to adding and subtracting rows of the matrix

Picturing Solutions

2 vars:

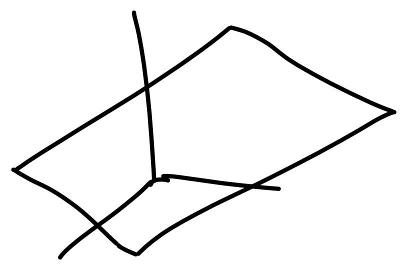
$x + 2y = 7$ is a line in $\mathbb{R}^2$

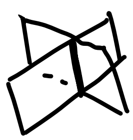
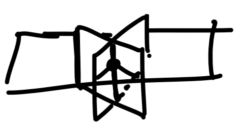
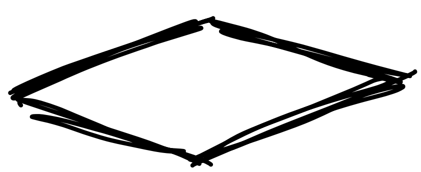


- solutions to a system of 2-var linear equations can be:
  - a point 
  - a line 
  - empty set 
  - all of $\mathbb{R}^2$ e.g. $5 = 5$

3 vars:

$x + 2y + 3z = 7$ is a plane in $\mathbb{R}^3$



- solutions to a system of 3-var linear equations can be:
  - a line 
  - all of $\mathbb{R}^3$ e.g. $0 = 0$
  - a point  (3 planes intersecting)
  - a plane 

1st goal for course: How to solve systems of linear equs?

<u>Basic Fact:</u> Given a soln to a system, it must be a soln to any multiple of any eqn, as well as the sum of any two eqns.

Ex: $x+2y=7$ R1 $\rightsquigarrow$ $0 \cdot R1$: $0=0$ ← we got extra solns!

$3x+2y=8$ R2 $\rightsquigarrow$ R2 : $3x+2y=8$

<u>Fix:</u> Do only reversible procedures to eqns to make sure we don't lose information

↓

3 operations (row operations)

- swap rows
- multiply a row by a nonzero constant
- add a multiple of 1 row to another (and replace the row you added to)

Ex:

$$\begin{aligned} R1 \quad x+2y=7 &\longrightarrow R1 : \quad x+2y=7 \\ R2 \quad 3x+2y=8 &\longrightarrow R2-3R1 : \quad -4y=-13 \end{aligned}$$

+3R1